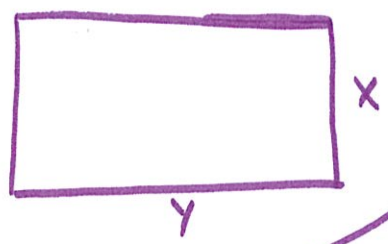
Mr. Michael T. Davis WLPCS Calculus	Section 5.4 Practice Problems April 24, 2018

Name: Solutions / Answers

3. *Maximizing Perimeter* What is the smallest perimeter possible for a rectangle whose area is 16 in^2, and what are its dimensions?

Minimize Perimeter

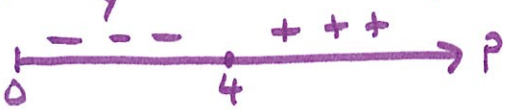


$P = 2x + 2y$ $\quad A = xy$

$P = 2\left(\frac{16}{y}\right) + 2y$ $\quad 16 = xy$

$P = \frac{32}{y} + 2y$ $\quad \frac{16}{y} = x$

$P'(y) = \frac{-32}{y^2} + 2$



$0 = \frac{-32}{y^2} + 2$

$\frac{32}{y^2} = 2$

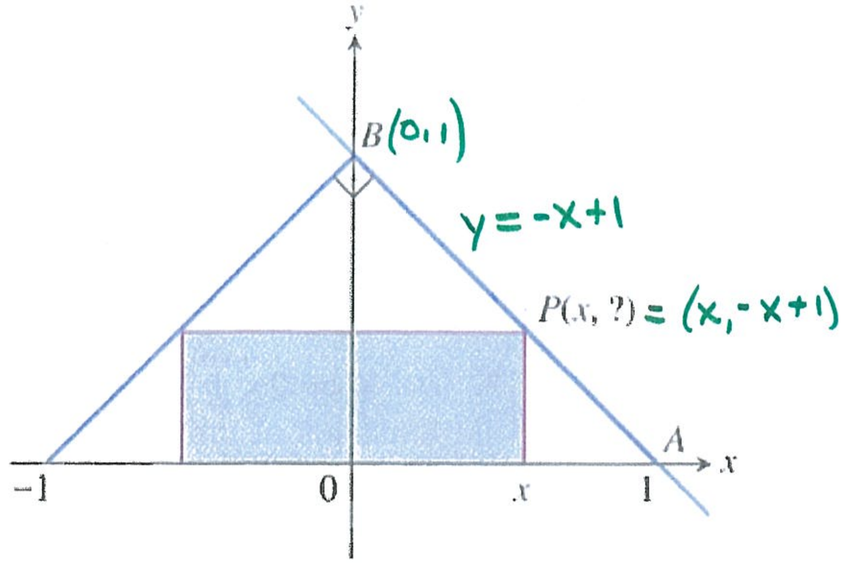
$16 = y^2$

$4 = y$

$x = 4$

The rectangle has smallest possible perimeter of 16 in with dimensions 4 in by 4 in since P' changes from Neg. to Pos.

5. *Inscribing Rectangles* The figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.



(a) Express the y-coordinate of P in terms of x. [*Hint:* Write an equation for the line AB.]

(b) Express the area of the rectangle in terms of x.

(c) What is the largest area the rectangle can have, and what are its dimensions?

a. the slope of $\overline{AB}$ is $m=-1$ since the coordinates of B are $B(0,1)$ and $A(1,0)$. An equation of the line $\overleftrightarrow{AB}$ is $y=-x+1$. The coordinates of P are $P(x,-x+1)$.

b. Area of the rectangle is $A(x) = \text{base} \cdot \text{height}$
$A(x) = 2x(-x+1) = -2x^2+2x$

c. Maximize Area $A'(x) = -4x+2$ $\quad 0=-4x+2$
$x=\frac{1}{2}$ C.P.

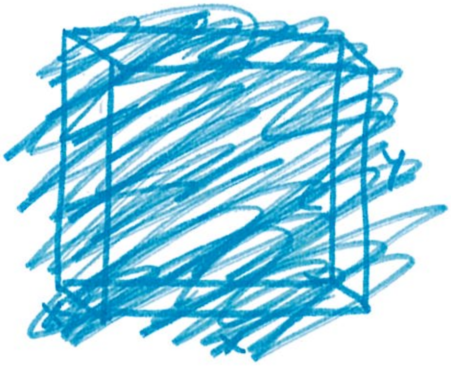
A': $+ + + +$ on $(0, \frac{1}{2})$, $- - - - -$ on $(\frac{1}{2}, 1)$

$A(\frac{1}{2}) = -2(\frac{1}{2})^2 + 2(\frac{1}{2}) = -2(\frac{1}{4}) + 1 = -\frac{1}{2}+1 = \frac{1}{2}\text{ in}^2$

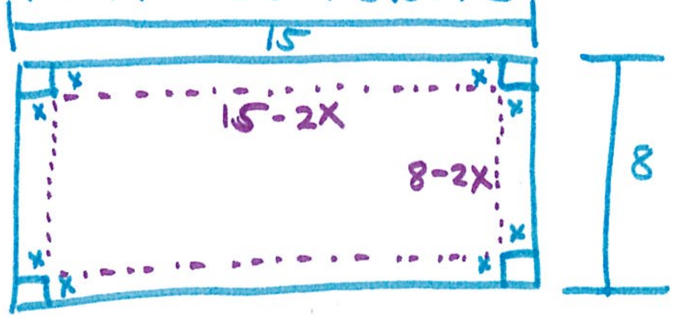
$A(x)$ is a max. of $A=\frac{1}{2}\text{ in}^2$ at $x=\frac{1}{2}$ since $A'(x)$ changes from Pos. to Neg. at $x=\frac{1}{2}$ in.

If $x=\frac{1}{2}$ then $y=-\frac{1}{2}+1=\frac{1}{2}$, so dimensions are 1 in by $\frac{1}{2}$ in

7. ***Optimal Dimensions*** You are planning to make an open rectangular box from an 8- by 15-in. piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is its volume?



Maximize Volume



$$V(x) = x(8-2x)(15-2x) = x(120-46x+4x^2)$$

$$V(x) = 120x - 46x^2 + 4x^3$$

$$V'(x) = 120 - 92x + 12x^2$$

$$0 = 120 - 92x + 12x^2$$

$$0 = 4(3x^2 - 23x + 30)$$

$$0 = 4(3x-5)(x-6)$$

$$x = \frac{5}{3} \text{ C.P.} \qquad x \neq 6$$

$$15 - 2\left(\frac{5}{3}\right) = 15 - 3\tfrac{1}{3} = 11\tfrac{2}{3}$$

$$8 - 2\left(\frac{5}{3}\right) = 8 - 3\tfrac{1}{3} = 4\tfrac{2}{3}$$

$+ + +$ $\quad - - - -$ V'

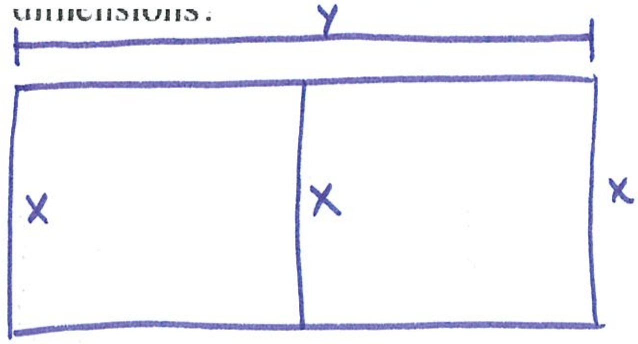
$0 \qquad \frac{5}{3}$

$$V\left(\frac{5}{3}\right) = 120\left(\frac{5}{3}\right) - 46\left(\frac{5}{3}\right)^2 + 4\left(\frac{5}{3}\right)^3 \approx 90.74 \text{ in}^3$$

$V(x)$ has a max. volume of $V \approx 90.74 \text{ in}^3$ at $x = \frac{5}{3}$ in since $V'(x)$ changes from Pos. to Neg at $x = \frac{5}{3}$.

The dimensions are $\frac{5}{3}$ in by $4\frac{2}{3}$ in by $11\frac{2}{3}$ in

10. *The Shortest Fence* A 216-m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?



Minimize Fence Length

$L(x,y) = 3x + 2y$

Area is 216 m^2 $\quad A(x,y) = xy \quad 216 = xy$

$y = \frac{216}{x} \quad L(x) = 3x + 2\left(\frac{216}{x}\right) = 3x + \frac{432}{x}$

$L'(x) = 3 + -\frac{432}{x^2} \quad 0 = 3 - \frac{432}{x^2} \quad \frac{432}{x^2} = 3$

$432 = 3x^2 \quad x^2 = 144 \quad x = 12 \quad y = \frac{216}{12} = 18$

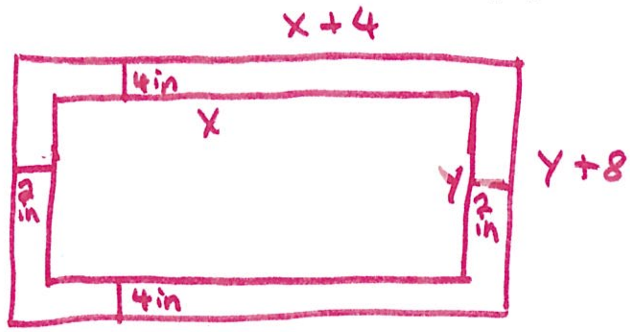
$\underset{0}{\vdash} - - - \underset{12}{\bullet} + + + \rightarrow L'$ $\quad L(12) = 3(12) + \frac{43}{12} = 36 + 36$

$= 72$

$L(x)$ is a min. length of $L = 72$ m at $x = 12$ m since $L'(x)$ changes sign from Neg. to Pos. at $x = 12$.

The outer rectangle has dimensions 18 m by 12 m

13. *Designing a Poster* You are designing a rectangular poster to contain 50 in^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin at each side. What overall dimensions will minimize the amount of paper used?



Printing Area $50 = xy$

overall Area of Paper $A = (x+4)(y+8)$

$50 = xy \quad y = \frac{50}{x} \qquad A(x) = (x+4)\left(\frac{50}{x}+8\right)$

$A(x) = 50 + 8x + \frac{200}{x} + 32 = 8x + \frac{200}{x} + 82$

$A'(x) = 8 + \frac{-200}{x^2} \qquad 0 = 8 - \frac{200}{x^2} \qquad \frac{200}{x^2} = 8$

$200 = 8x^2 \quad 25 = x^2 \quad x = 5$ in $\quad y = \frac{50}{5} = 10$ in

The paper used (Area) is a min. of $(5+4)(10+8)$ or 360 in^2 with dimensions 9in by 18in overall since $A'(x)$ changes sign from Neg. to Pos at $x = 5$in.